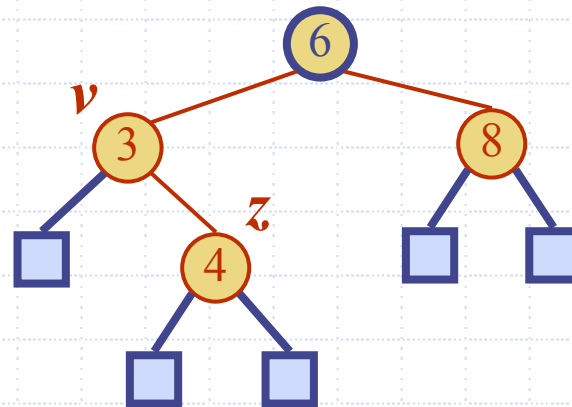


Red-Black Trees

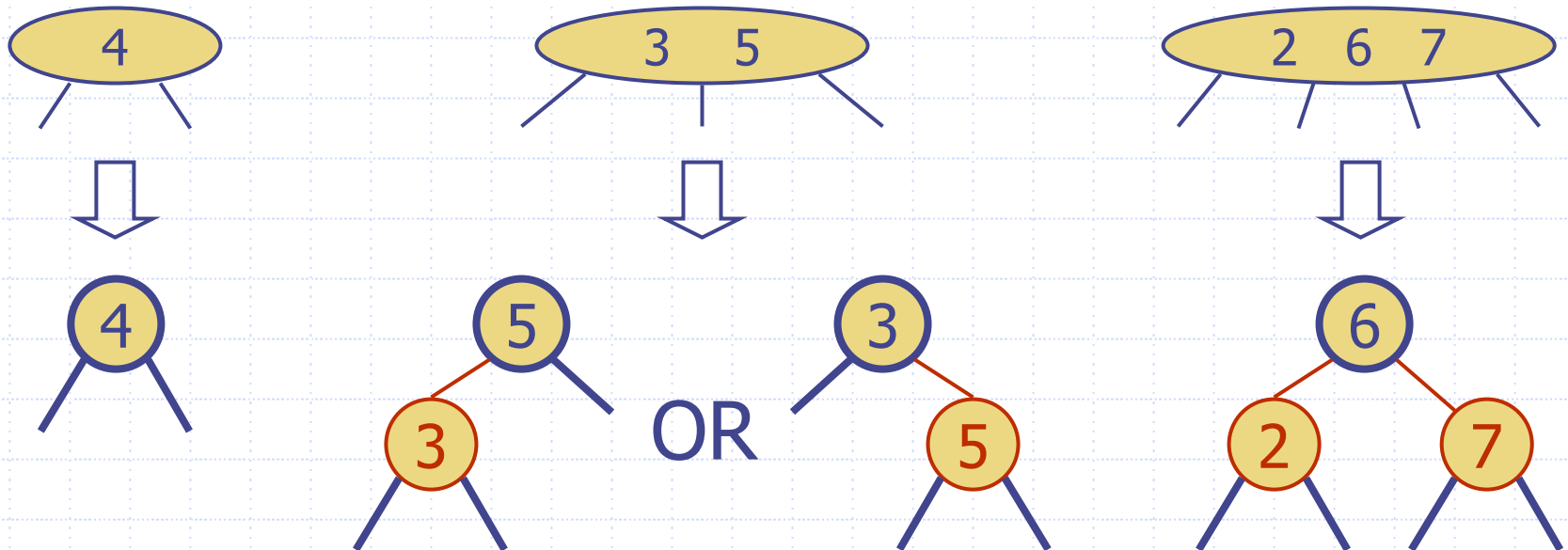


Outline and Reading

- ◆ From (2,4) trees to red-black trees (§9.5)
- ◆ Red-black tree (§9.5)
 - Definition
 - Height
 - Insertion
 - ◆ restructuring
 - ◆ recoloring
 - Deletion
 - ◆ restructuring
 - ◆ recoloring
 - ◆ adjustment

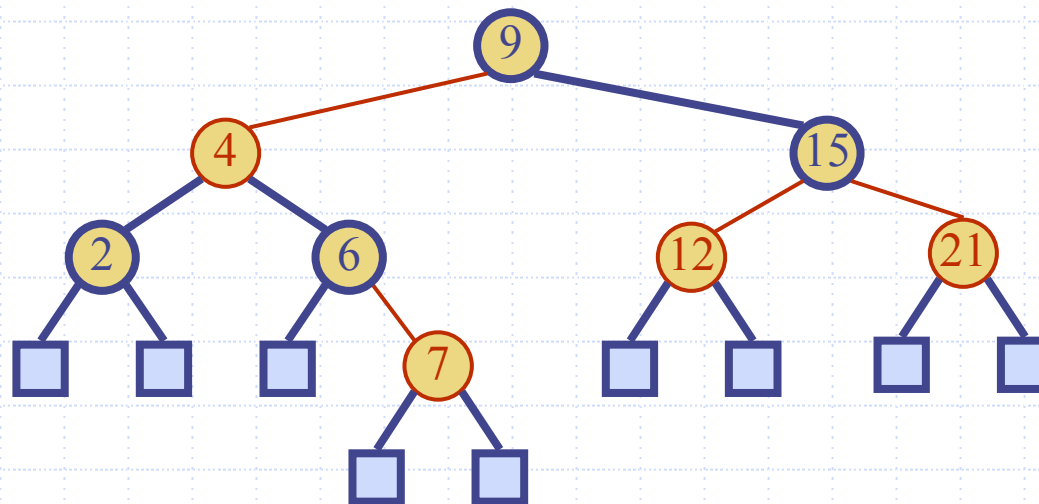
From (2,4) to Red-Black Trees

- ◆ A red-black tree is a representation of a (2,4) tree by means of a binary tree whose nodes are colored **red** or **black**
- ◆ In comparison with its associated (2,4) tree, a red-black tree has
 - same logarithmic time performance
 - simpler implementation with a single node type



Red-Black Tree

- ◆ A red-black tree can also be defined as a binary search tree that satisfies the following properties:
 - **Root Property:** the root is black
 - **External Property:** every leaf is black
 - **Internal Property:** the children of a red node are black
 - **Depth Property:** all the leaves have the same black depth



Height of a Red-Black Tree

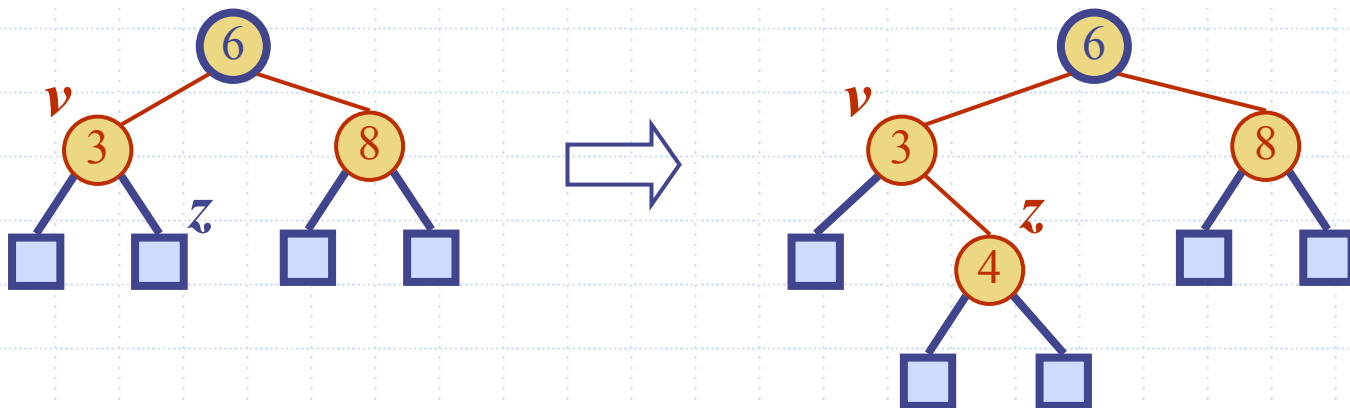
◆ **Theorem:** A red-black tree storing n items has height $O(\log n)$

Proof:

- The height of a red-black tree is at most twice the height of its associated (2,4) tree, which is $O(\log n)$
- ◆ The search algorithm for a binary search tree is the same as that for a binary search tree
- ◆ By the above theorem, searching in a red-black tree takes $O(\log n)$ time

Insertion

- ◆ To perform operation `insertItem(k, o)`, we execute the insertion algorithm for binary search trees and color **red** the newly inserted node z unless it is the root
 - We preserve the root, external, and depth properties
 - If the parent v of z is black, we also preserve the internal property and we are done
 - Else (v is red) we have a **double red** (i.e., a violation of the internal property), which requires a reorganization of the tree
- ◆ Example where the insertion of 4 causes a double red:

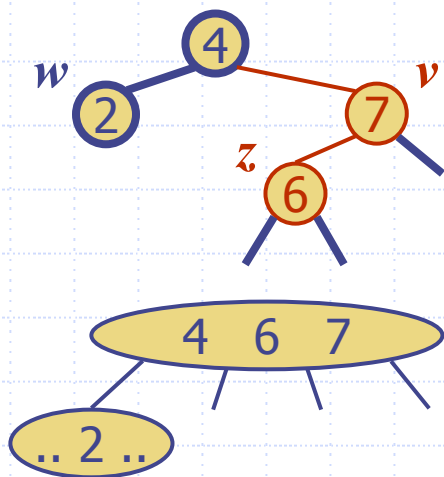


Remedying a Double Red

- ◆ Consider a double red with child z and parent v , and let w be the sibling of v

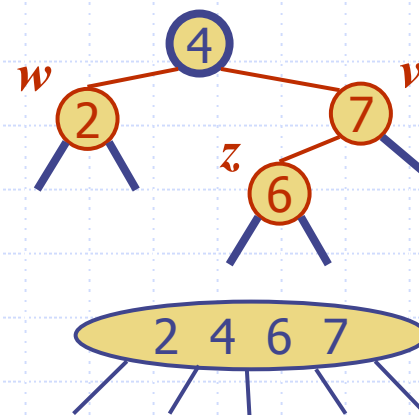
Case 1: w is black

- The double red is an incorrect replacement of a 4-node
- **Restructuring**: we change the 4-node replacement



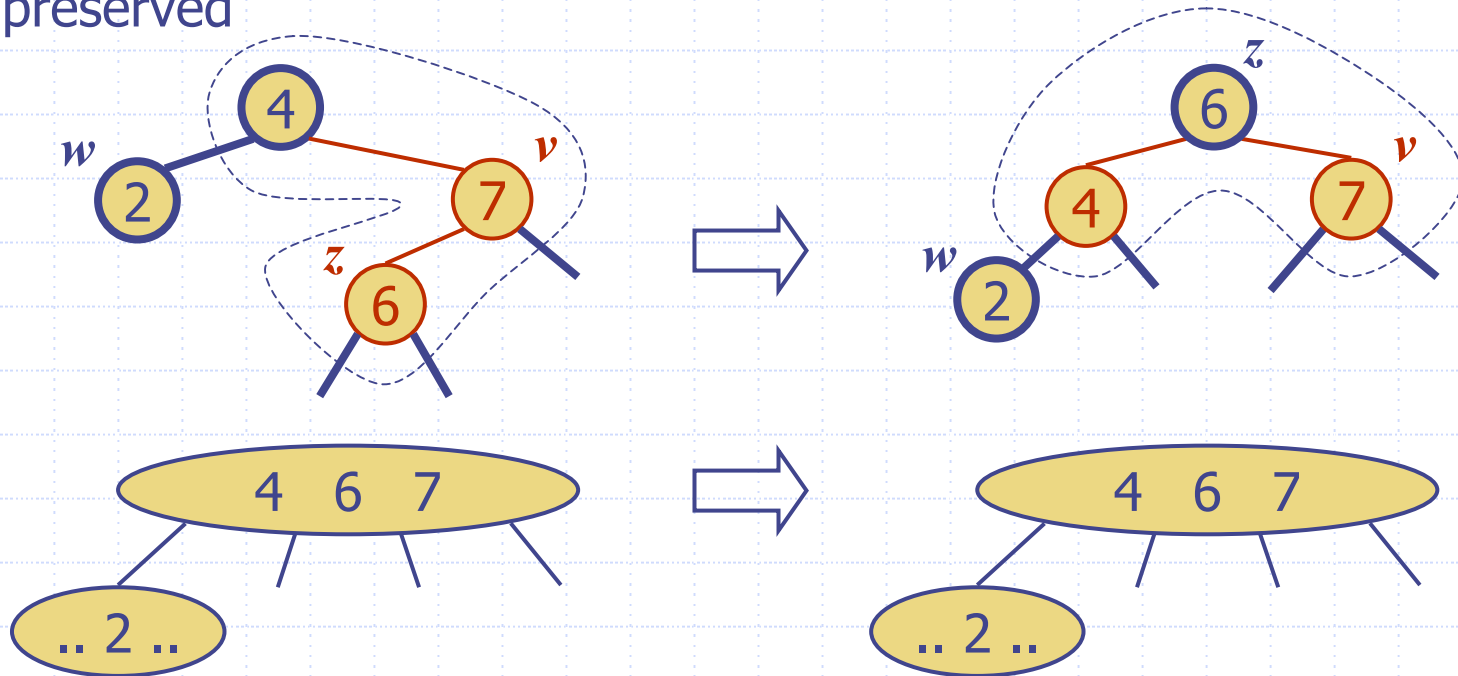
Case 2: w is red

- The double red corresponds to an overflow
- **Recoloring**: we perform the equivalent of a **split**



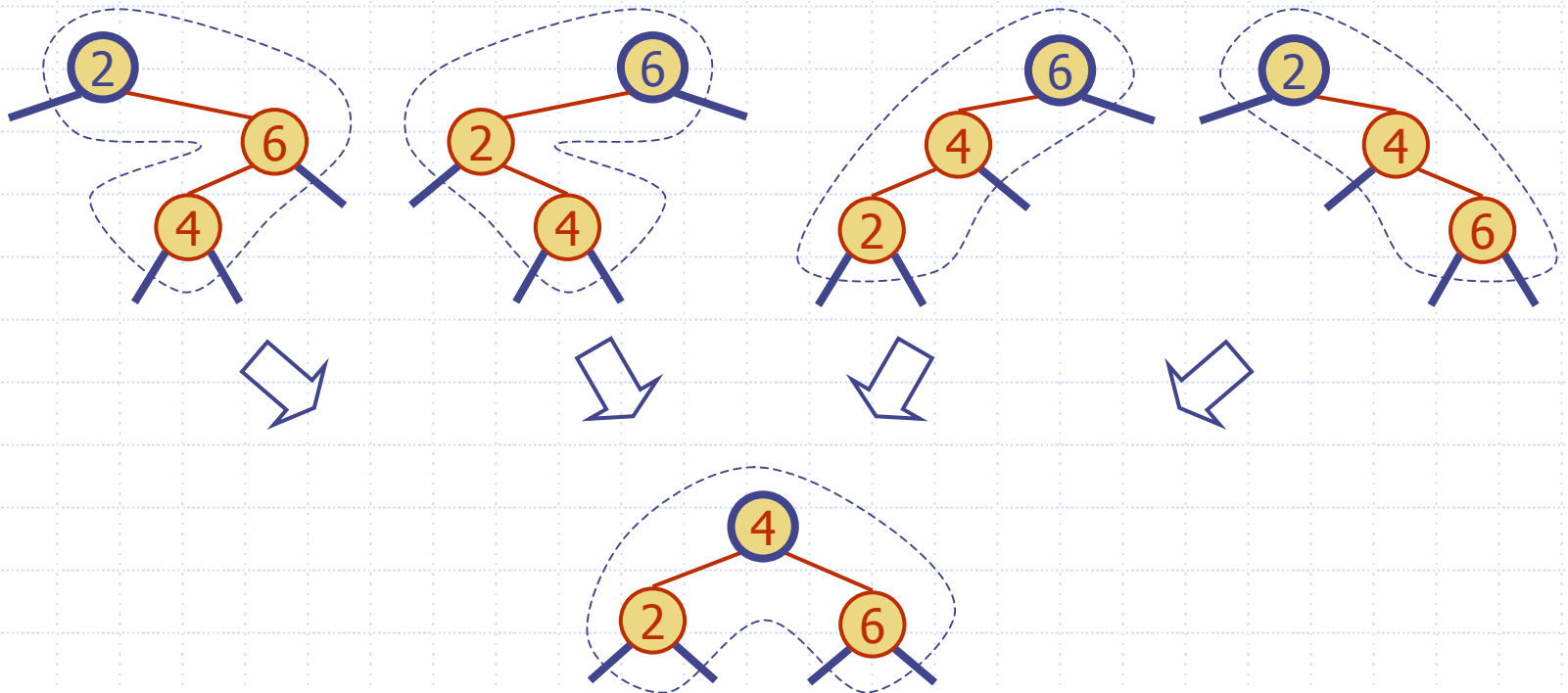
Restructuring

- ◆ A restructuring remedies a child-parent double red when the parent red node has a black sibling
- ◆ It is equivalent to restoring the correct replacement of a 4-node
- ◆ The internal property is restored and the other properties are preserved



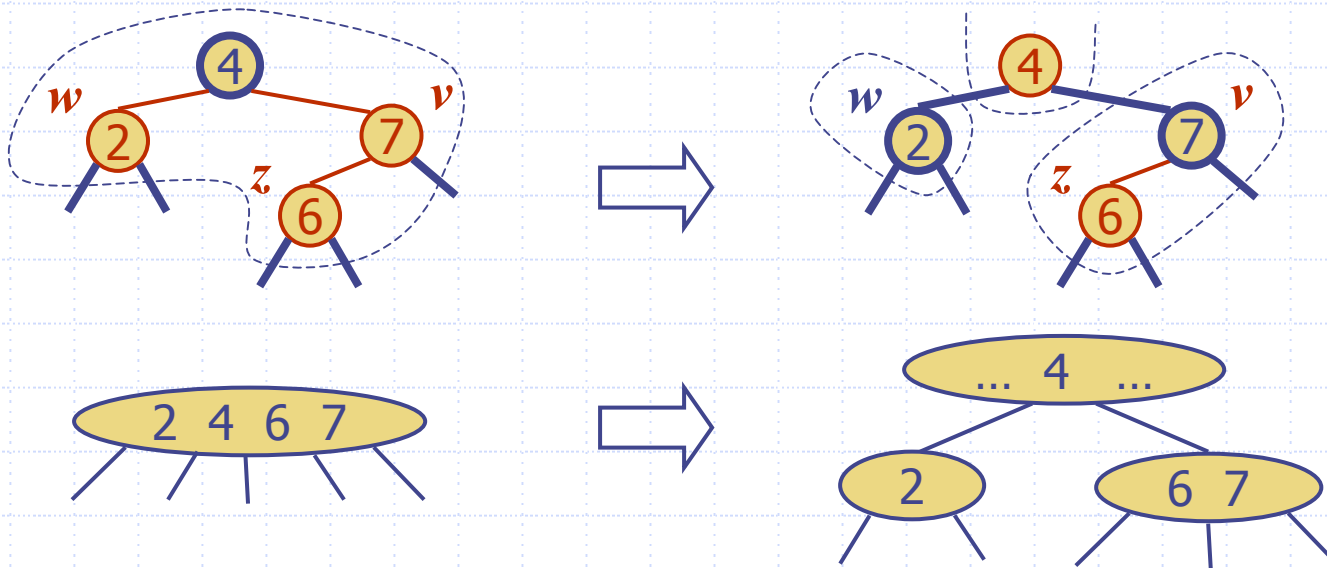
Restructuring (cont.)

- ◆ There are four restructuring configurations depending on whether the double red nodes are left or right children



Recoloring

- ◆ A recoloring remedies a child-parent double red when the parent red node has a red sibling
- ◆ The parent v and its sibling w become black and the grandparent u becomes red, unless it is the root
- ◆ It is equivalent to performing a split on a 5-node
- ◆ The double red violation may propagate to the grandparent u



Analysis of Insertion

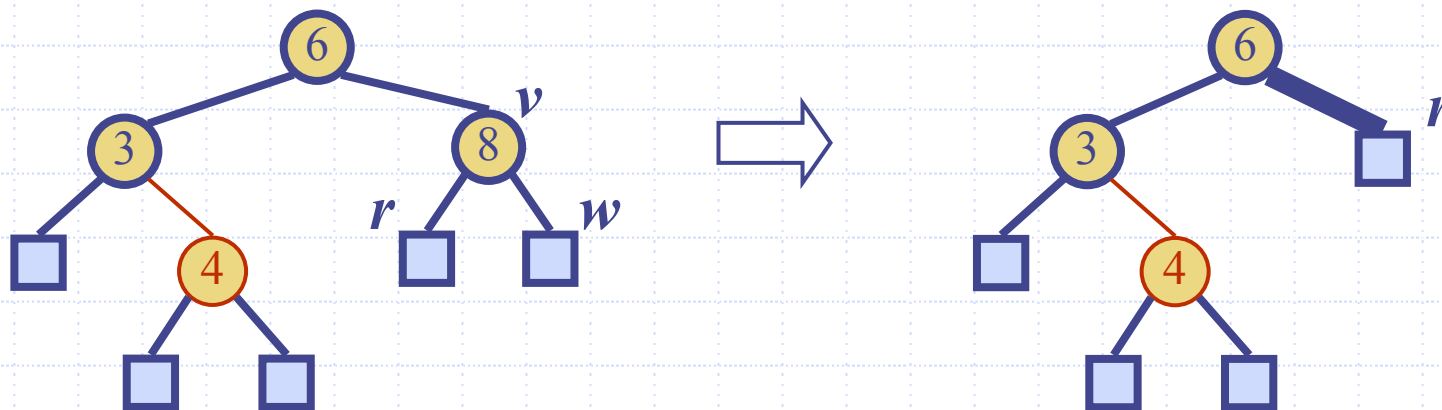
Algorithm *insertItem(k, o)*

1. We search for key k to locate the insertion node z
2. We add the new item (k, o) at node z and color z red
3. **while** *doubleRed*(z)
 if *isBlack*(*sibling*(*parent*(z)))
 $z \leftarrow \text{restructure}(z)$
 return
 else { *sibling*(*parent*(z)) is red }
 $z \leftarrow \text{recolor}(z)$

- ◆ Recall that a red-black tree has $O(\log n)$ height
- ◆ Step 1 takes $O(\log n)$ time because we visit $O(\log n)$ nodes
- ◆ Step 2 takes $O(1)$ time
- ◆ Step 3 takes $O(\log n)$ time because we perform
 - $O(\log n)$ recolorings, each taking $O(1)$ time, and
 - at most one restructuring taking $O(1)$ time
- ◆ Thus, an insertion in a red-black tree takes $O(\log n)$ time

Deletion

- ◆ To perform operation `remove(k)`, we first execute the deletion algorithm for binary search trees
- ◆ Let v be the internal node removed, w the external node removed, and r the sibling of w
 - If either v or r was red, we color r black and we are done
 - Else (v and r were both black) we color r **double black**, which is a violation of the internal property requiring a reorganization of the tree
- ◆ Example where the deletion of 8 causes a double black:



Remedying a Double Black

- ◆ The algorithm for remedying a double black node w with sibling y considers three cases

Case 1: y is black and has a red child

- We perform a **restructuring**, equivalent to a **transfer**, and we are done

Case 2: y is black and its children are both black

- We perform a **recoloring**, equivalent to a **fusion**, which may propagate up the double black violation

Case 3: y is red

- We perform an **adjustment**, equivalent to choosing a different representation of a 3-node, after which either Case 1 or Case 2 applies

- ◆ Deletion in a red-black tree takes $O(\log n)$ time

Red-Black Tree Reorganization

Insertion		remedy double red
Red-black tree action	(2,4) tree action	result
restructuring	change of 4-node representation	double red removed
recoloring	split	double red removed or propagated up

Deletion		remedy double black
Red-black tree action	(2,4) tree action	result
restructuring	transfer	double black removed
recoloring	fusion	double black removed or propagated up
adjustment	change of 3-node representation	restructuring or recoloring follows